

# Set membership techniques in FTC schemes

{Florin.Stoican, Sorin.Olaru}@supelec.fr



SUPELEC Systems Sciences (E3S) - Automatic Control Department, Gif sur Yvette, France

## Preliminaries

Fault detection and isolation (FDI) strategies that employ set membership testings have been described in [1] and permit a robust fault detection of sensor faults.

This is realized through sets that describe a signal of interest which characterize the functioning regime for a sensor (healthy or faulty mode).

## Theoretical tools

In a practical setting, polytopes are used for set construction.

Enhancements studied take into account particular cases:

- zonotopes (hypercube projections on a reduced space) – [2]
- star shaped domains (particular cases of nonconvex regions)

The viability theory [3] offers a general framework for the various set theoretical and differential inclusion notions.

## Advantages

This scheme offers:

- robust fault detection, on a comparable level with classical sensor fusion schemes in terms of performance
- computational flexibility (the sets are computed offline and only set membership testings are executed online)
- stability and invariance guarantees – [4]

## Current research

The extensions to nonconvex perturbations and LPV systems is under development.

The plant nonlinearities and their impact on the overall scheme is to be analysed as well as the presence of actuator faults.

The enhancements of the necessary tools include: tighter approximations of the mRPI sets and particular cases of convex / nonconvex sets.

## References

- [1] M. M. Seron, X. W. Zhuo, J. A. De Doná, and J. Martinez, "Multisensor switching control strategy with fault tolerance guarantees," *Automatica*, vol. 44, no. 1, pp. 88–97, 2008.
- [2] F. Stoican, S. Olaru, J. A. De Doná, and M. M. Seron, "Zonotopic ultimate bounds for linear systems with bounded disturbances," in *Proceedings of the 18th IFAC World Congress*, Milano, Italy, 28 August–2 September 2011, pp. 9224–9229.
- [3] J. P. Aubin, *Viability theory*. Birkhauser, 1991.
- [4] F. Stoican, S. Olaru, M. M. Seron, and J. A. De Doná, "A fault tolerant control scheme based on sensor switching and dwell time," in *Proceedings of the 49th IEEE Conference on Decision and Control*, Atlanta, Georgia, USA, 15–17 December 2010.
- [5] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, *Diagnosis and fault-tolerant control*. Springer, 2006.
- [6] F. Stoican, S. Olaru, J. A. De Doná, and M. M. Seron, "Improvements in the sensor recovery mechanism for a multisensor control scheme," in *Proceedings of the 29th American Control Conference*, Baltimore, Maryland, USA, 30 June–2 July 2010, pp. 4052–4057.

## Assumptions

- abrupt faults
- the reference signal has an offset relative to the origin
- *well-posedness*: the poles of the plant and estimators can be chosen through pole placement methods

## Fault scenarios

For the moment only sensor outages discussed:

- total output outages

$$y_i = C_i x + \eta_i \xrightarrow{\text{FAULT}} y_i = 0 \cdot x + \eta_i^F$$

$$y_i = C_i x + \eta_i \xleftarrow{\text{RECOVERY}} y_i = 0 \cdot x + \eta_i^F$$

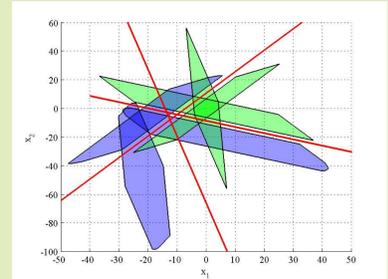
- generic fault scenarios (a signature matrix for each type of fault)

$$y_i = \Pi_i [C_i x + \eta_i] + [I - \Pi_i] \eta_i^F$$

## FDI mechanism

A robust FDI mechanism permits accurate fault detection by analyzing the appartenance of a residual signal – [5]:

$$r_i = \hat{z}_i^{UP} - (I - M_i C_i) \hat{z}_i$$



$$R_i^H \cap R_i^F = \emptyset$$

For the further use of a previously faulty sensor a recovery mechanism has to be employed – [6]:

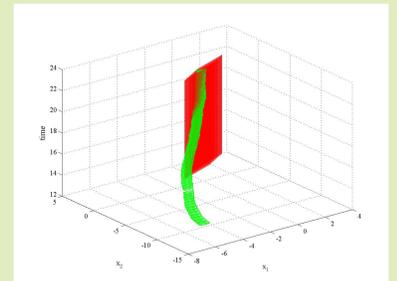
$$Z_{I_H}^j = (I - M_j C_j)^{-1} [\{-\hat{z}_j^{UP}\} \oplus M_j N_j \oplus I_{I_H}]$$

necessary:

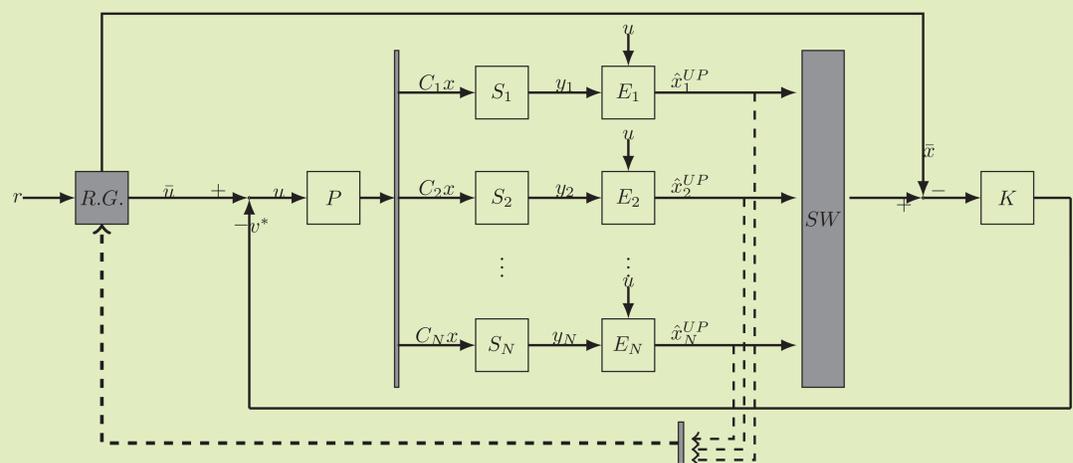
$$Z_{I_H}^j \cap \tilde{S}_j \neq \emptyset$$

sufficient:

$$Z_{I_H}^j \subset \tilde{S}_j$$



## FTC scheme



Scheme classification:

- LTI system:  $x^+ = Ax + Bu + w$
- multiple actuators:  $x^+ = Ax + B_i u + w$
- nonlinear system:  $x^+ = Ax + \gamma(x) + w$

FDI signals:

- estimation:  $\hat{x}_i^+ = (A - L_i C_i) \hat{x}_i + B u_i + L_i y_i$
- residual:  $r_i = \hat{z}_i^{UP} - (I - M_i C_i) \hat{z}_i$

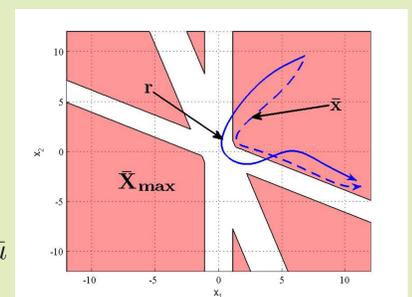
Reference governor:

$$\min \|r - \bar{x}\|$$

subject to

$$\bar{x} \in \bar{X}$$

$$\bar{x}^+ = A \bar{x} + B \bar{u}$$



Control strategies:

- switch between healthy sensors
- convex hull of healthy sensors

## Example of functioning

