FAULT TOLERANT CONTROL BASED ON SET-THEORETIC METHODS

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Assumptions

Assumptions necessary for an exact fault tolerant control (FTC) scheme:

- controllability and observability
- abrupt faults (e.g., in sensor output)
- redundancy of the scheme (e.g., failure of one sensor does not make the scheme unobservable)
- persistence of excitation (e.g., through an offset in reference signals)
- boundedness of noises and perturbations

Various faults scenarios can be accommodated:

- total output outages

$$y_i = C_i x + \eta_i \xrightarrow{\text{FAULT}} y_i = 0 \cdot x + \eta_i^F$$
$$y_i = C_i x + \eta_i \xleftarrow{\text{RECOVERY}} y_i = 0 \cdot x + \eta_i^F$$

- generic fault scenarios (a signature matrix for each type of fault)

$$y_i = \Pi_i \left[C_i x + \eta_i\right] + \left[I - \Pi_i\right] \eta_i^F$$

Theoretical tools

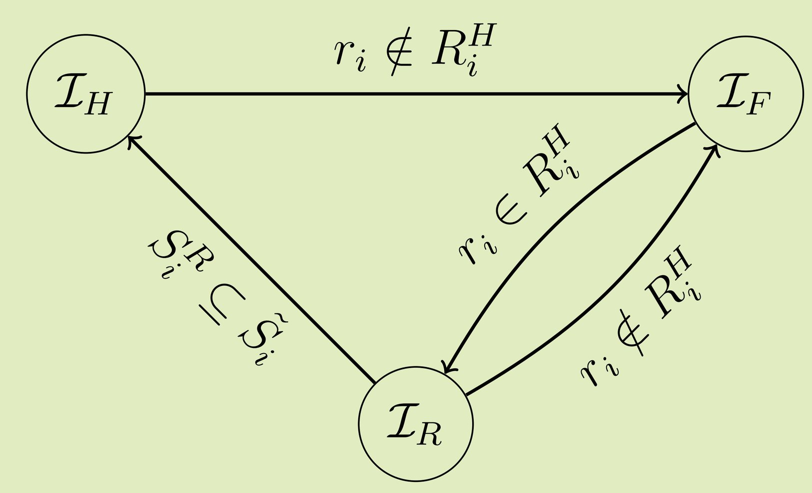
Following results in [1, 2], set strategies are used to describe regions which characterize healthy and faulty functioning for the sub-systems of interest.

Set-theoretic tools:

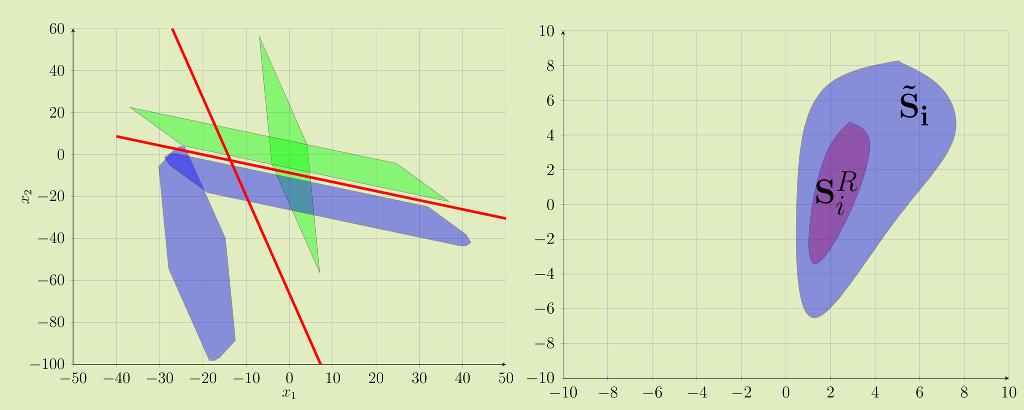
- invariance notions (ultimate bounds, RPI and mRPI sets, recheability, etc) for set characterization [2]
- inclusion time computation for convergence and set inclusions
- various families of sets (polytopes, zonotopes, star-shaped regions) which allow extensions to non-convex perturbations and nonlinear/LPV systems
- mixed integer programming and hyperplane arrangements for descriptions of non-convex regions and subsequent optimizations [3]
- dwell-time and cyclic invariance for switched systems [4]

FDI mechanism

An exact FDI mechanism which transitions between *healthy*, *faulty* and *under recovery* functioning modes via set membership testings [7]:



- residual inclusion into healthy/faulty sets ($r_i \in R_i^H / R_i^F$) with exactness guaranteed for $R_i^H \cap R_i^F = \emptyset$
- necessary and sufficient conditions for recovery ($S_i^R \cap \tilde{S}_i \neq \emptyset$ and $S_i^R \subseteq \tilde{S}_i$)



Advantages

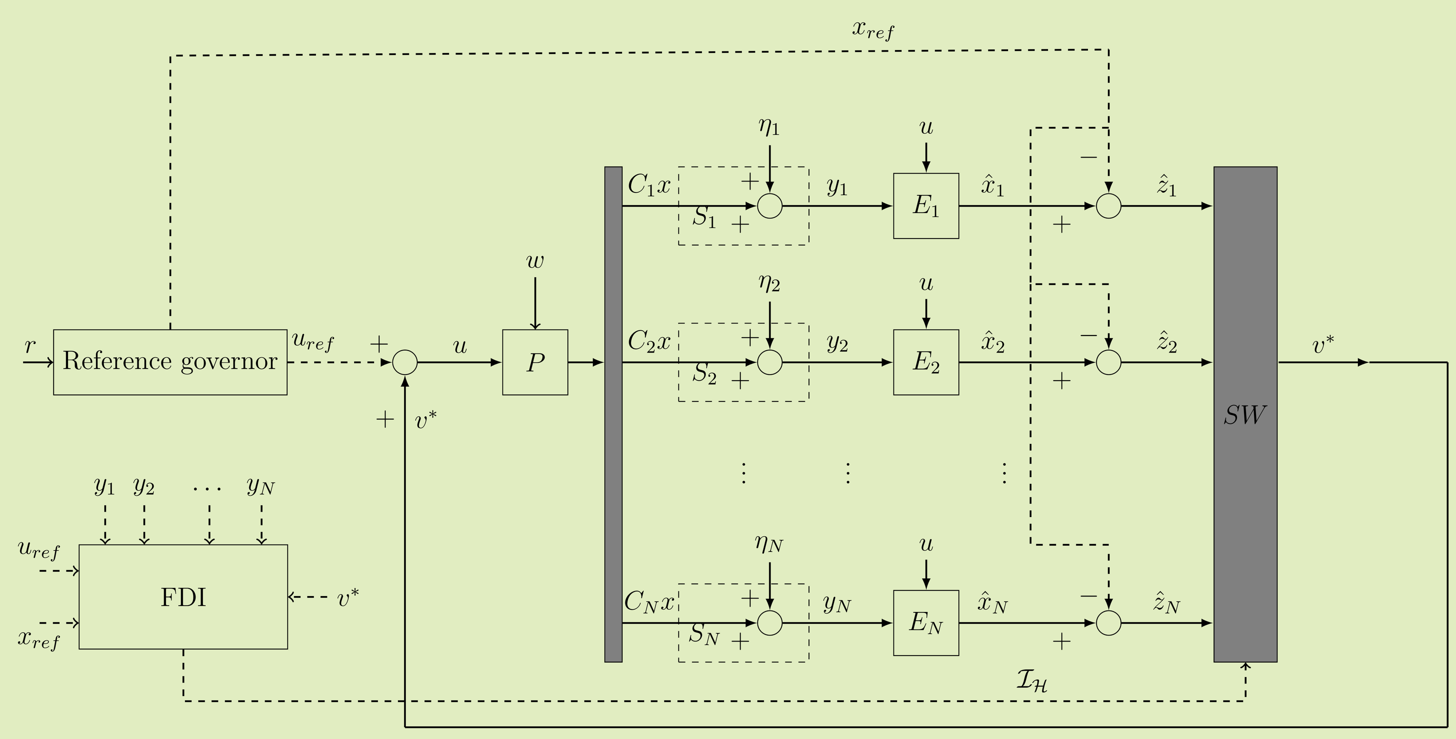
This scheme offers [5]:

- exact FDI, stability and invariance guarantees [2, 6] and performance comparable with classical sensor fusion schemes
- reduced computational demands (the sets are computed offline and only set membership testings are executed online)
- a compromise between complexity of representation and numerical accuracy
- extensions to various cases of dynamics

FTC scheme

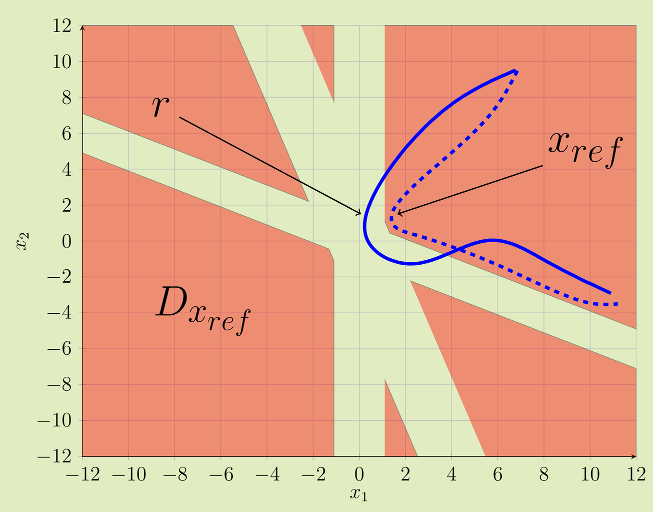
Scheme classifications:

- **LTI**/switched/with delay or nonlinear systems
- **multi-sensor**/actuator
- implicit/**explicit FDI**
- passive/**active FTC**



Residual design:

- output measurement $r_i = y_i - C_i x_{ref}$
- estimation: $r_i = \hat{x}_i$
- moving finite horizon: $r_i = f(u, u^-, \ldots, y_i, y_i^-, \ldots)$



Residual design and separation conditions provide feasible regions for references and/or control variables:

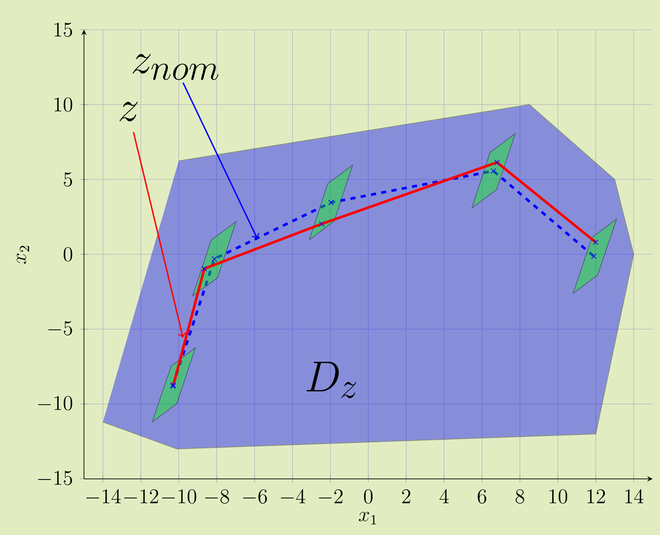
$$\begin{aligned} r_i^H &= C_i z + \eta_i \\ r_i^F &= -C_i x_{ref} + \eta_i^F. \end{aligned} \quad \Rightarrow \quad \left(\{C_i z\} \oplus N_i\right) \cap \left(\{-C_i x_{ref}\} \oplus N_i^F\right) = \emptyset$$

which leads to the feasible region:

$$\mathbb{D}_{ref} = \{(z, x_{ref}) : \text{ separation holds } \forall i \in \mathcal{I}\} \quad \Rightarrow \quad \begin{cases} \mathbb{D}_{x_{ref}} = \{x_{ref} : (S_z, x_{ref}) \subseteq \mathbb{D}_{ref}\} \\ \mathbb{D}_{x_{ref}} = \{z : (z, X_{ref}) \subseteq \mathbb{D}_{ref}\} \end{cases}$$

Control strategies:

- active FTC with fix gain feedback
  - controlled invariance through fix gain design [6]
  - reference governor synthesis for exact FDI [8]
- active FTC with MPC
- passive FTC



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